

Numerical Examples of Evolutionary Processes

[English translation of “*Esempi Numerici di Processi di Evoluzione*” (1954)]*

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Abstract. Some consequences of the Darwinian principle of evolution by survival of the fittest are analyzed. Considering that this principle reduces evolution to a purely statistical phenomenon, we draw the conclusion that it may apply not only to living organisms but also to elements of any kind able to reproduce and to undergo hereditary changes (mutations). By applying the Darwinian principle to the most primitive elements with the required properties, viruses or artificially constructed elements (for instance, numbers), we show that the Darwinian principle alone is not sufficient to explain the origin of an evolutionary process like the biological one. At least one additional principle is needed in order to explain the origin of such an evolutionary process. We suggest that the theory of gene symbiosis may serve as this complementary principle. By using numerical elements with reproduction rules that make symbiosis (or utilitarian association) necessary, we show that evolutionary processes with promising properties are likely to arise. In the last part of the paper, we describe an evolutionary process obtained with numerical elements during a series of experiments performed by the electronic computer at the Institute for Advanced Study, Princeton, NJ, USA, in the spring of 1953.

Keywords: history of artificial life, computational evolution, symbiogenesis, symbioorganisms, digital organisms, open-ended evolution

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Preface by the translator and editor

This is an English translation of Nils Aall Barricelli's 1954 publication *Esempi Numerici di Processi di Evoluzione*. The original paper was published in the Italian journal *Methodos* (Barricelli, 1954). To the best of our knowledge, it is the first publication describing experiments in computational evolution and artificial life.

The initial translation was completed by Giulia Bianchi, and the final version was arrived at collaboratively through an iterative process of review and editing by both Bianchi and Tim Taylor. Our aim has been to balance fidelity to the original text with readability in contemporary English.

The abstract of this translation is a lightly-edited version of the original English-language summary provided by Barricelli at the end of the original paper (Barricelli, 1954, p.68). The legibility of some of the figures in the original paper was poor, so we have recreated all of them here based upon the originals. Furthermore, the original paper did not provide captions for the figures; in this translation we have included captions based upon those provided by Barricelli in a later paper in which the same figures were presented (Barricelli, 1962).

Our accompanying commentary provides further details on the translation process, and also presents an extensive discussion of the paper's content, the context in which it was written, its relationship to Barricelli's earlier and later works, and the legacy and significance of the work today (Taylor & Bianchi, 2026).

Giulia Bianchi & Tim Taylor

1 The problem of the origin of living organisms

As science has gradually enabled us to understand and explain an ever-increasing number of natural phenomena, the questions that remain unresolved have become of ever-increasing interest to scholars around the world. One of the questions that has most aroused human curiosity can be stated as follows:

“Why do living organisms exist?”

A fundamental contribution toward the solution of this problem was provided by Darwin. Although subsequent research has led to modifications of Darwin’s concepts of how hereditary changes occur, the core principle discovered by Darwin stands firm: species evolve through hereditary changes and natural selection. The most vital organisms, best adapted to their environment, survive. As a result, the hereditary changes most beneficial for individuals spread, while the most harmful disappear. Species thus have the possibility to evolve toward ever greater perfection, developing organs and characteristics that make them increasingly adapted to life and to competition with other species.

Nevertheless, this theory doesn’t answer our question—it doesn’t tell us why living organisms exist; however, it is an attempt to answer another question which is not without importance in this regard, namely: “Why are living organisms so complex, and why are they equipped with such efficient and perfect organs and faculties, so useful to the life of the organisms themselves?”

Once this last question is answered, the existence of living organisms no longer seems so inexplicable—the problem no longer seems so arduous. What is truly astonishing and what at first glance seems inexplicable about living organisms is their complexity, their efficiency, and their perfection. Once one finds a way to interpret these facts, what remains seems to boil down to something much simpler: “how did the first primitive cell (e.g., the first amoeba) originate?” Everything else seems to be explainable based upon the principles of Darwin’s theory.

It is not surprising that since Darwin’s time to the present day, many circles have maintained an often unwarranted optimism about the possibility of clarifying “this final detail” to achieve a complete theory on the nature of living organisms.

But a question that might embarrass optimists is the following:

“If it were so simple to create living organisms, why don’t you create some yourselves?”

It is not difficult to find the words to answer this question. But as time goes by, the situation becomes increasingly embarrassing, and one feels the need for something more than words to counter such objections.

2 Criticism of the Darwinian evolution principle

Darwin's theory of evolution through heredity and natural selection reduces evolution to a purely statistical phenomenon. At first glance, the properties that must be assigned to the elements of a statistical population to enable a process of evolution based upon the principles of Darwin's theory seem spectacularly simple. To have an evolutionary process based upon the survival of the fittest elements, it is sufficient that these elements have:

1. the ability to reproduce, and
2. the ability to undergo hereditary changes.

There are many examples of elements with the ability to reproduce in nature, and it is not difficult to define and construct new ones artificially.

It is well known, for example, that there exist certain chemical compounds, called autocatalysts, that catalyze chemical reactions between other compounds, a reaction whose result is, among other things, an increase of the catalyst itself. Such an autocatalyst will therefore become more and more abundant as the reaction proceeds, or in other words, it reproduces itself through a process called autocatalysis. At the same time, some compounds (substrates) originally present are consumed, while other compounds (by-products) are produced together with the autocatalyst through the same chemical reaction.

Also in atomic physics, autocatalytic phenomena similar to the one described above are well known. For instance, a neutron starting a chain reaction by hitting a Uranium U_{235} nucleus and causing fission, resulting in the production of two or more neutrons among other things, is reproducing itself by a process that has the characteristics of what may be considered an atomic autocatalysis. The Uranium U_{235} atoms act as the substrates of the nuclear reaction, which, besides neutrons also produces some "by-products," i.e. elements of atomic weight roughly half the weight of Uranium.

Artificially, it is very easy to construct or merely define elements, for instance of arithmetic nature, with the ability to reproduce. If desired, one may also choose the definition so as to avoid any concern for factors that are not conceptually indispensable, such as substrates and by-products.

For instance, one may use as self-reproducing elements a group of numbers written in the first row of a cross-section paper—see Figure 1 where negative numbers are underlined—and one may choose arbitrarily a reproduction rule for these numbers. The reproduction rule used in Figure 1 is the following: in one time unit (generation), a positive number m is reproduced m squares to the right, and a negative number n is reproduced $-n$ squares to the left. The result obtained from the first row by following this reproduction rule is recorded in the second row. Applying the same reproduction rule to the second row, one obtains the third row, and so on. Of course, to continue, one will also need to choose some rules to define what should happen when two numbers land in the same square. This will be discussed later on.

[illegible]

Figure 1: Self-reproducing numbers (see text). In this and all other figures, negative numbers are shown as underlined. This simple example exhibits adaptive selection but no extensive evolution phenomena are possible.

The other property essential for Darwinian evolution, the ability to undergo hereditary changes, is not as common as the ability to reproduce, but elements with this capacity are known to exist, and there is reason to believe it is based on chemical or physical phenomena. The elements in question are viruses and genes. In both cases, these are albumin molecules with autocatalytic power and the capacity to undergo hereditary changes (mutations). This is the fundamental ability, from an evolutionary perspective, that differentiates these organic autocatalysts from the many other autocatalysts widely known in inorganic chemistry. The similarities between genes and viruses are quite remarkable from both a chemical and biological perspective, and, as we will see, there is reason to believe that they are the same type of elements.

Artificially, there is no difficulty in defining elements that, in addition to the ability to reproduce, also have the ability to undergo hereditary changes. In the numerical elements described above, one may for instance choose some mutation rules utilizing the cases where two numbers land in the same square. The number to be placed in the square in this case might be different from both the colliding numbers and will thus represent a mutation.

The mutation rule adopted in Figure 1 is as follows: two numbers that land in the same square are added together, and from their sum, the number in the square above is subtracted (unless the square above the collision is empty); if three numbers land in the same square, they are added together, and from their sum twice the number in the square above is subtracted (unless that square is empty, in which case nothing will be subtracted) and so on, each time a new number is added, the number contained in the square above is subtracted once more.

In this way, we have a class of numerical elements that can both reproduce and undergo mutations. The conditions necessary for an evolutionary process based on the principles of Darwin's theory would thus be present. The numbers with the highest probability of survival in the environment created in Figure 1 by the rules we have chosen will live on. The rest will be gradually eliminated. A process of adaptation to the environmental conditions, that is, a process of Darwinian evolution, will take place.

This example of Darwinian evolution clearly demonstrates that something is still missing in order to explain the formation of organs and faculties as complex as those of living organisms. No matter how many mutations occur, the numbers in Figure 1 will remain numbers forever. They will never become living organisms!

One may object that the situation is different when it comes to viruses and genes, due to the complexity of the albumin molecules, which might be expected to present far greater evolutionary potential.

We can certainly state that if the situation is different for viruses and genes, it must be for fundamentally different reasons, but certainly not because of their evolutionary possibilities. Most genes are known to have only two allelic states and it is very uncommon for a gene to possess more than 10 allelic states. A gene with 10 allelic states can only exist in 10 different varieties, attainable through one or more successive mutations. The evolutionary possibilities

of such a gene are therefore extremely limited. Selection can do nothing other than choose the most viable of the 10 possible allelic states. The situation is roughly the same for simple viruses (those lacking sexual reproduction).

It is not impossible that genes with a much greater number of allelic states might be found, and it is plausible that the number of allelic states of a gene is typically greater than that capable of producing viable organisms (the only ones that can be experimentally discovered). But the consensus among biochemists is that an albumin molecule, no matter how many mutations it undergoes, will never be anything more than an albumin molecule. There is no risk of it developing into a “Homo sapiens.”

Regarding the possibility of evolution, the situation does not appear to be any better for albumin molecules than for the numbers in Figure 1. The main difference that can be observed is that “the numbers written by nature” can have a somewhat more complicated appearance.

However, if the principle of Darwin’s theory is supposed to explain the complexity and efficiency achieved by living organisms, it must be possible to apply it to even the simplest elements capable of reproduction and mutation, namely, genes and viruses. One cannot start with elements that already possess remarkable complexity and efficiency, such as cells, even if relatively simple unicellular organisms such as amoebae are chosen. Obviously, amoebae have far greater complexity, efficiency, and ability to adapt than the numbers in Figure 1. But these features are among the properties that should be explained by the theory of evolution rather than being taken as given.

If, instead of the numbers in Figure 1, to initiate an evolutionary process like that of living organisms, it were necessary to invent elements capable of assuming extremely diverse forms and of a complexity and efficiency comparable to that of living organisms, then the explanation of the properties of these elements, even after a more or less prolonged evolutionary period, should not be sought in Darwin’s theory. Rather, it should be asked of the inventor who constructed such elements.

In conclusion, the principle of Darwin’s theory is insufficient to explain the evolution of living organisms starting from elements with only the capabilities of reproduction and mutation. At least one more theoretical principle is needed to explain how these elements could give rise to organisms with capacities for variation and evolution comparable to those of living organisms.

3 The theory of gene symbiosis

The solution to this problem was found by taking inspiration from the surprising similarities that exist between viruses and genes. In both cases these are albumin molecules of the same type, endowed with autocatalytic power and the capacity for hereditary changes. Especially among plasmagenes, all intermediate states between genes and viruses exist, so much so that

in many cases it is unclear whether one is dealing with a plasmagene or a virus living in symbiosis with the cell.

A bold hypothesis was proposed by Kozo-Polyansky in 1924, when some understanding of these analogies was already starting to develop. Kozo-Polyansky's hypothesis is that all genes and also various other molecules of the cell that are equipped with autocatalytic mechanisms descend from corresponding viruses or virus-like organisms that, through symbiosis, gradually become associated with the rest of the cell.

According to this theory, which Kozo-Polyansky calls the "syntrophogenesis theory," the evolutionary process that allowed the formation of the cell was initiated by a symbiosis between certain organisms similar in nature to viruses. Over time, these would then be joined by new symbionts of the same nature and in ever-increasing numbers, and the evolutionary process thus initiated would have made possible the formation of the various organs of the cell.

This theory solves the dilemma we faced when trying to push the principle of Darwin's theory to its ultimate consequences.

Even if each gene has a very limited number of allelic states, often no more than 2, by associating a large number n of genes (or viruses) in a single organism, the number of allelic states can become very large, that is, 2^n or more. Among all these allelic states, natural selection undoubtedly has a good selection to choose from. If the number n of genes is a few thousand, as in *Drosophila*, it's no longer so fantastic to imagine that among these 2^n combinations there could be many corresponding to organisms as complex as *Drosophila* or even *Homo sapiens*.

A significant achievement of the theory of gene symbiosis is that it has made it possible to understand the origin of gene recombination and therefore of sexual reproduction and, very importantly, it has made it possible to predict the existence of viruses with sexual reproduction.

Indeed, if genes were associated through symbiosis, they possessed from the very beginning the characteristic ability of all parasites and symbionts to change hosts, that is, to associate with different individuals of the same species. This ability, from the very beginning, gave gene association the rapidity of adaptation characteristic of sexually reproducing organisms, based on the fact that a large number of useful mutations, one for each gene, can simultaneously spread throughout the species without interfering with each other, simultaneously solving their respective phenotypic problems.

According to the theory of symbiosis, sexual reproduction would be the result of a gradual improvement of this ability to replace and recombine genes, which has existed since the beginning in composite organisms created by the symbiosis of multiple genes.

An equally important consequence is that the initial result of symbiosis must have been the first small piece of an original chromosome that would then, little by little, create the other organs of the cell. This primordial chromosome must have had characteristics resembling those of a virus with sexual reproduction.

In other words, the theory of symbiosis allowed us to predict the existence of viruses with sexual reproduction.

The discovery of sexual reproduction in bacterial viruses (bacteriophages) is an important confirmation of the theory.

To test the theory of gene symbiosis, we can once again make use of numerical elements. It will suffice to modify the reproduction rules employed in Figure 1 to favor some form of utilitarian association (symbiosis) between different numerical elements. This will allow us to see whether it is true that by associating elements with the basic properties described above, we can initiate an evolutionary process that develops based on the same mechanism that governs biological evolution.

4 The utilitarian association of numerical elements

To promote utilitarian association (symbiosis) we can make the following modifications to the reproduction rules adopted in Figure 1:

- a) A number n is not repeated in the square below as in Figure 1, but only n squares to the right in the row below if it is positive or $-n$ squares to the left in the row below if it is negative (see Figures 2 and 3). We express this by saying that the number n moves to square (n) in the next row without repeating itself.
- b) If in the new position (n) the number lands under a square occupied by another number m different from n then a second number n will be placed in position (m) that is m squares to the right if m is positive and $-m$ squares to the left if m is negative, calculating the squares starting from the original position of the number n but in the row below.

4																	
			4														
						4											
									4								
													4				

Figure 2: Action of basic symbiosis rule on single positive number (see text).

In this way, a number n can reproduce itself when another number m different from n is present, see Figure 4. If, while reproducing itself, it happens to land several times under squares occupied by different numbers, the same number n can also reproduce itself multiple times, see Figure 5 where the number 4 in the second row reappears three times in the next row.

											<u>2</u>
									<u>2</u>		
							<u>2</u>				
					<u>2</u>						
			<u>2</u>								
	<u>2</u>										
<u>2</u>											

Figure 3: Action of basic symbiosis rule on single negative number (see text).

							4									<u>3</u>
										4				<u>3</u>		
								4				<u>3</u>		<u>4</u>		
					4				<u>3</u>			<u>4</u>				
		4				<u>3</u>			<u>4</u>					4		
4			<u>3</u>			<u>4</u>					4					
	<u>3</u>		<u>4</u>						4					4		
						4							4			
									4							4
													4			

Figure 4: Reproduction with the basic symbiosis rule (see text).

4					3				<u>2</u>	
				4			<u>2</u>	3		
		4			<u>2</u>		4	4		
			<u>2</u>			4				
	<u>2</u>									4

Figure 5: Multiple reproduction with the basic symbiosis rule (see text).

These reproduction rules are intended to make reproduction of a numerical element possible only when different numerical elements are present. In this way, the utilitarian association (symbiosis) of different numerical elements becomes necessary in order to make reproduction possible.

To the above reproduction rules, one may add arbitrary mutation rules, for example by exploiting instances where two numbers collide in the same square. In one of these rules, which we will call the “exclusion norm,” two or more numerical elements that collide in the same square are deleted (excluded) if they are not identical. The square in which exclusion has occurred is marked with an “x” (exclusion sign). If all elements that collide in the same square are identical, one of them remains.

In this way, no mutations occur and the exclusion norm cannot therefore allow for evolution unless it is combined with other norms capable of providing the necessary mutations. In Figure 6, the absence of mutations is clearly manifested by the fact that the original elements 5, 1, and -3 are present everywhere and no new element appears.

		5			5		1	<u>3</u>	1	<u>3</u>							
			5		1	<u>3</u>	x	<u>3</u>	1		x	<u>3</u>		<u>3</u>			
5		1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>				
<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>		<u>3</u>	
		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>		5
<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	<u>3</u>
	<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>
	5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1	5	<u>3</u>
1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>
<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1
5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>
<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>		<u>3</u>	1		5	<u>3</u>	1	<u>3</u>	<u>3</u>

Figure 6: Formation of a numerical organism (see text).

On the other hand, stable organisms can easily form through the organization of the existing numerical elements. In Figure 6, after a pair of exclusions and a short period of adaptation, a species (5, -3, 1, -3, □, -3, 1, □)¹ emerged that can be transplanted without difficulty, proving capable of reproducing without changes, except in the event of collisions with other numerical elements. In Figure 7 this species has been transplanted, and one can clearly see how reproduction occurs. The species, formed as a result of the symbiosis of the three genes 5, 1, and -3, offers a simple illustration of the way in which a species with multiple genes can originate according to symbiosis theory.

¹*Editors' note:* In the original paper this species is printed as (5, -3, 1, 3, x, -3, 1, x), where the fourth number is erroneously shown as positive 3 and the “x” symbol is used to indicate an empty square rather than an exclusion sign. To avoid confusion, here and throughout this translation we use the “□” symbol to indicate an empty square, and reserve the “x” symbol solely to indicate an exclusion sign.

[illegible]

Figure 7: Transplantation and reproduction characteristics of the numerical organism shown in Figure 6 (see text).

Organisms like this, born from the symbiosis of multiple elements, often have the ability to exchange elements with other varieties of similar constitution. According to symbiosis theory, this ability is to be considered as the origin of sexual reproduction, and the gene exchanges that occur indeed exhibit surprising similarities with crosses between two varieties of a haploid species.

Two varieties particularly suited to illustrate this primitive form of cross-breeding are (9, -11, 1, -7, 9, -11, 1, -7, ... periodic) and (5, -11, 1, -3, 5, -11, 1, -3, ... periodic).² The second variety can be considered to have been obtained from the first through two mutations that have changed element 9 into a 5 and element -7 into a -3. We may decide to consider the first variety as the wild type and the second as the double mutant. These two varieties have the advantage over the others of giving rise, through gene recombination, only to viable varieties. In Figure 8 these two varieties are crossed by transplanting the first (wild type) on the left and the second (double mutant) on the right. As can be seen in the figure, the variety (5, -11, 1, -7, 5, -11, 1, -7, ... periodic), obtained as a result of the cross, has inherited only one of the two mutations, namely, element 5 instead of 9. As it spread, this variety has destroyed one of the parental organisms, the one on the left (wild type). The parental organism on the right (the double mutant) also loses ground to the new variety and would be gradually eliminated if Figure 8 were continued.³ The varieties were deliberately chosen so that the resulting cross is in a favorable position compared to the parental organisms in order to obtain viable crosses.

In Figure 9 the same parental organisms are arranged in reverse order, the double mutant on the left and the wild type on the right. From this crossing we get the organism (9, -11, 1, -3, 9, -11, 1, -3, ... periodic) that has inherited the other mutation, namely, element -3 instead of -7. Also in Figure 9, the crossing result has an advantage over the two parental organisms, both of which would be eliminated if the figure were continued.

If we now cross the two hybrids (that is, the results of the two crossings) in different ways, we can recover the parental organisms, according to the same inheritance laws that apply to

²*Editors' note:* In the original paper the periodic varieties shown in this section are printed without commas between the numbers and without the elipsis (. . .). We have added these elements to maintain a consistent format throughout the paper, adopting the style that Barricelli used in sections 5 and 6.

³*Editors' note:* In the original paper this sentence erroneously refers to Figure 9 rather than Figure 8.

[illegible]

Figure 8: Cross-breeding in numerical organisms, wild type on left and double mutant on right (see text).

[illegible]

Figure 9: Cross-breeding in numerical organisms, double mutant on left and wild type on right (see text).

haploid organisms. In Figure 10, by using the second hybrid on the left and the first to the right, we obtain the double mutant as a result, and in Figure 11, using the same parental organisms in reverse order, we recover the wild type.

It is now clear that the exclusion norm exhibits many promising properties. However, the absence of mutations that characterizes this norm must be addressed if we want to have possibilities for evolution based not only on recombinations of pre-existing elements but also on new elements arising from mutations.

5 Application of mutations in numerical organisms

A suitable source of mutations can be obtained in several ways. The approach we will employ at first is a modification of the exclusion norm that will be called the “conditional norm.” The only difference is that the x (exclusion sign) must in some cases, according to a specific mutation law, be replaced by a numerical element (mutation). The chosen mutation law is:

1. An x (exclusion sign) that falls under a square occupied by a number remains.
2. An x that falls under an empty square (or under another x) is replaced by a number equal to the distance between the nearest number to the left and the nearest number to the right of the aforementioned empty square (the distance is calculated in number of squares). If the two said numbers have the same sign, then the new number (distance) will take a positive sign, otherwise a negative sign.
3. If there is no number to the right or left of the said empty square (for example, because the square is located near the edge of the sheet) then the x remains.

For example, in Figure 12, in the second row where the two numbers 3 and -4 collide, there will not be an x but a +5 because the distance between the two numbers 3 and 2 in the top row is 5 squares and because those numbers have the same sign. On the other hand, in Figure 13 where the two numbers -2 and -5 collide, an x will remain since there is no number to the left of -2. Similarly, in Figure 14 the x will remain where the two numbers 3 and -4 collide because the square above is not empty.

This “conditional norm” offers the advantage of creating an adequate number of mutations while not departing too far from the previous norm.

With the mutation law now chosen, the organisms we crossed in Figures 8, 9, 10 and 11 can reproduce and cross in much the same way. While some mutations might arise during crossing, this usually does not lead to considerable changes. In some cases, however, for example in Figure 10, it may be necessary to slightly modify the initial conditions in order to avoid a series of mutations that would otherwise prevent normal crossbreeding.

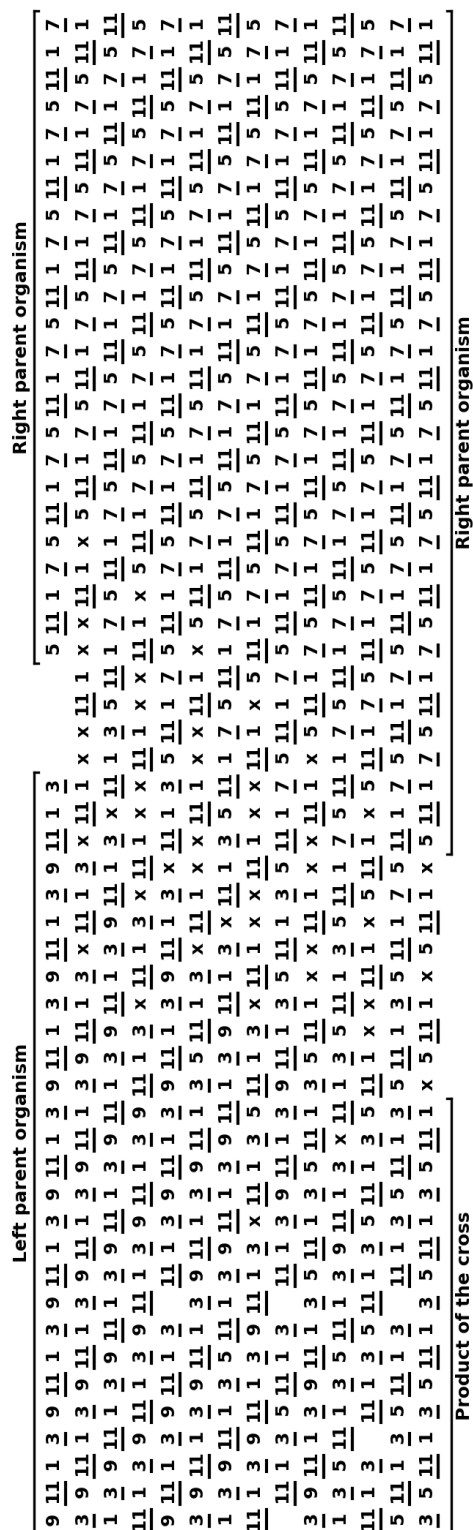


Figure 10: Cross-breeding of hybrids to recover the double mutant (see text).

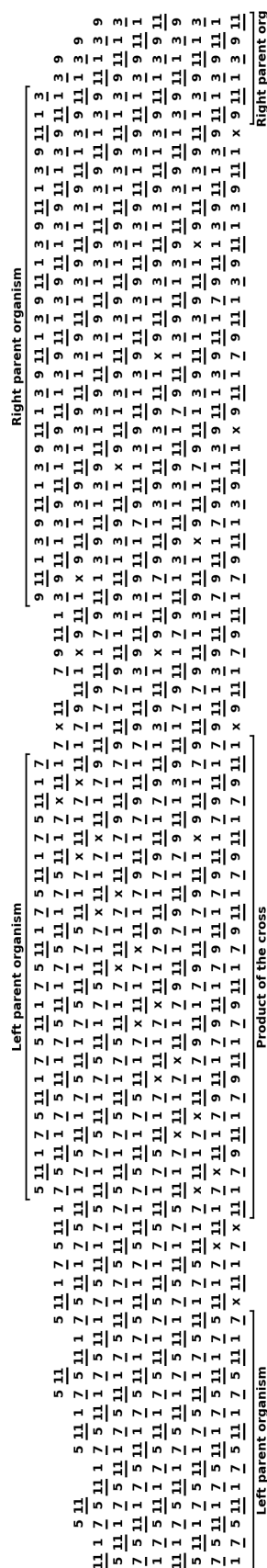


Figure 11: Cross-breeding of hybrids to recover the wild type (see text).

3					2		<u>4</u>				
	2		5				<u>2</u>				
			2			2		5	2		
					2			2			2
							2			2	
									2		
											2

Figure 12: Mutation with the conditional norm (see text).

		<u>2</u>			<u>5</u>
x					

Figure 13: Mutation with the conditional norm (see text).

3			2			<u>4</u>
		3	x	2		

Figure 14: Mutation with the conditional norm (see text).

6 Origin and competition of numerical organisms

We have seen how the utilitarian association (symbiosis) of numerical elements (genes) can generate “numerical organisms” with promising properties. By replacing some genes of a numerical organism, we can obtain organisms with very different properties and, as we know, the varieties that can be obtained in this way are astronomical in number, exactly as in terrestrial life forms.

But all the organisms shown thus far are artificial constructs, and it is unclear how they would manage in competition with other organisms. It would be interesting instead to see if, by choosing random numerical elements in the first row, one could observe in the subsequent rows some forms of organization of the elements into numerical organisms capable of reproducing themselves and competing for their place in the universe we have defined. Such an experiment was conducted in Figure 15, where the first row (first generation) contains only the numbers 1 and -1 chosen by a random extraction method (playing heads or tails with two coins and noting 1 whenever two heads came up, -1 whenever two tails came up, and leaving an empty square whenever one head and one tail came up).⁴ As expected, the species

⁴*Editors’ note:* While preparing our commentary on this paper we discovered some errors in Figure 15 as it appeared in the original paper. We have shown the original version with errors in this translated paper. We discuss the errors and show a corrected version of the figure in our commentary (Taylor & Bianchi, 2026). After extensive examination, described in the commentary, we conclude that Barricelli’s subsequent discussion and interpretation of the phenomena shown in this figure remain valid for around 20% of experiments conducted under similar conditions from a randomly initialised state. The remaining 80% of experiments exhibit less interesting dynamics, settling into static or simple repeating states.

(1, -1, ... periodic) initially appeared in several regions. This species reproduces as shown in Figure 16. Shortly after, at the right end of the figure, the species (1, -2, 1, 1, -2, □, 1, -2, 1, 1, -2, □, ... periodic) emerged. This species is unable to reproduce on its own as seen in Figure 17, but lives as a parasite, or one might say, as a predator upon the species (1, -1, ... periodic), as shown in Figure 18.⁵ In Figure 17 we can see that the species gradually loses its elements and dies, but in Figure 18 we can see that as a parasite of the species (1, -1, ... periodic) it is capable of reproducing without undergoing changes as long as something of the host organism is left.

Further down, towards the center of Figure 15, a new species appears (4, □, □, □, 4, □, □, □, ... periodic) that can reproduce on its own as demonstrated in Figure 19. Further to the left follows the species (1, -3, 1, -3, 1, -3, ... periodic) which is also capable of independent reproduction as shown in Figure 20. In the middle of the figure and at approximately the same generation emerges the species (3, □, □, -3, □, □, ... periodic) which likewise reproduces as shown in Figure 21. The species (2, 2, -2, -2, ... periodic) appears between these two shortly thereafter and reproduces independently as illustrated in Figure 22.

The species shown in Figure 15 are very primitive and have not shown any instances of evolution.

7 The Princeton experiments. Evolutionary processes.

To obtain more complex organisms, the experiment conducted in Figure 15 must be repeated on a much larger scale.⁶ In practice this is possible only by using large computing machines.

Creating more complex organisms would also provide a greater chance of observing some evolutionary process. The evolution of numerical organisms is indeed possible, as with living organisms, through hereditary changes (for example, the replacement of some numerical gene) and selection.

Thanks to the generous contribution provided by the Norwegian scientific research fund “Norges Almenvidenskabelige Forskningsråd” and the opportunity offered by the “Institute for Advanced Study” of Princeton (NJ, USA) to use the electronic computer of that institute, I was able to conduct numerical experiments on a much greater scale than that depicted in Figure 15. Instead of 100 rows (generations) with 80 numbers each, the longest experiment conducted at Princeton ran for 5,400 generations with 512 numbers each.

The first row (generation) was populated with randomly drawn numbers. Different extraction methods were employed in different regions. For the most part we used the numbers +1, 0,

⁵*Editors' note:* In the original paper, in this sentence and the next, the host species is written as “(1, -1, ...)”. We have added the word “periodic” here to keep the format consistent with the rest of the paper.

⁶*Editors' note:* In the original paper, this paragraph, and the third paragraph of this section, erroneously refer to Figure 16 rather than Figure 15.

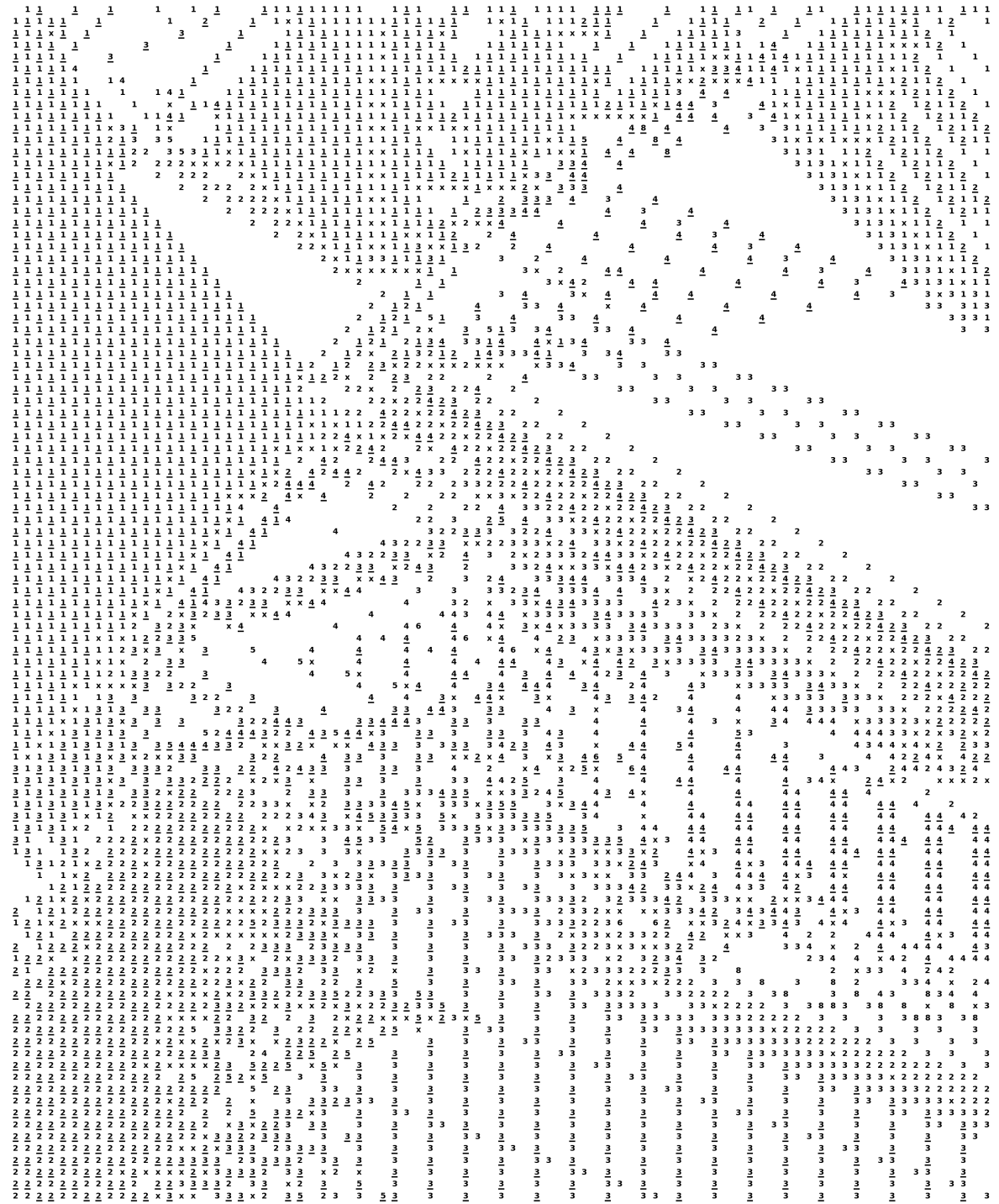


Figure 15: Spontaneous formation of numerical organisms in an experiment started with random numbers (see text).

					<u>1</u>	<u>1</u>					
				<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>				
			<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>			
		<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>		
	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>

Figure 16: Reproduction of a numerical organism that arose in the experiment of Figure 15 (see text).

and -1, where 0 denotes an empty square. However, in one region, larger numbers were also used.

It quickly became apparent that, by employing the same reproduction and mutation rules throughout the entire “Universe,” evolution led to homogeneity within a few hundred generations. In other words, either the whole “Universe” under study became occupied by a single species with identical individuals (lacking variability) or there was a complete disorganization of the entire “Universe.” In both cases, evolution ceased.

In order to prevent homogeneity, it was necessary to divide the “Universe” into different regions, each with its own set of rules. It proved sufficient to vary the mutation rules, while the reproduction rules stayed the same everywhere. With this expedient, it was possible to avoid homogeneity for many generations. Several experiments were performed by modifying the rules used and the number and dimensions of the regions.

Various evolutionary processes lasting several hundred generations were observed. In the most interesting case, the species undergoing evolution exhibited more than 20 different varieties, though not consecutive ones. A large portion of the varieties coexisted simultaneously, and in any given location, only 5 or 6 consecutive varieties could be found. The result provided detailed information on how the evolutionary process unfolded. Knowing the mutation rules in different regions allows us to observe how the hereditary changes that produce new varieties arise, how the most viable varieties spread at the expense of pre-existing ones, and the importance of recombinations of numerical genes (crosses of pre-existing varieties) in the production of new, more viable varieties.

An evolutionary process like the one just described does not always proceed uninterrupted over many generations. More often than not, the process halts when the species reaches a so-called “relative maximum of fitness,” that is, when a variety appears that can only give rise to less viable varieties with a single mutation, so that at least two mutations are needed to obtain more viable organisms. Two such mutations rarely appear together, and evolution, after a relative maximum of fitness is reached, may undergo a very long pause. But fitness can change, for example, as a result of changes in competing species that help determine the environment. A relative maximum of fitness can under certain conditions cease to be such, and evolution can begin again.

					1	<u>2</u>	1	1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>				
			1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>			
	1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		
<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>	
1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>
	1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1	<u>2</u>	1	1
<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1		1
1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1	
	1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>		1
<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>	
1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1	<u>2</u>
	1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>	1	1
<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1		1
1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1			1	
	1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1			1			1
<u>2</u>		1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1			1			1		
1	<u>2</u>		1	<u>2</u>		1	<u>2</u>		1			1			1			1	
	1	<u>2</u>		1	<u>2</u>		1			1			1			1			1
<u>2</u>		1	<u>2</u>		1			1			1			1			1		
1	<u>2</u>		1		1			1			1			1			1		
	1			1				1			1			1			1		1
		1			1			1			1			1			1		
			1			1			1			1			1			1	
				1			1			1			1			1			1
					1			1			1			1			1		
						1			1			1			1			1	
							1			1			1			1			1
								1			1			1			1		
									1			1			1			1	
										1			1			1			1
											1			1			1		
												1			1			1	
													1			1			1
														1			1		
															1			1	
																1			
																	1		
																		1	

Figure 17: Reproduction of another numerical organism that arose in the experiment of Figure 15. This species does not reproduce successfully when alone (see text).

						<u>1</u>	<u>1</u>			<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>					
					<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>					
				<u>1</u>	<u>1</u>	<u>1</u>	x	x	x	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>				
			<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>		
		<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	x	x	x	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	x	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>
<u>1</u>	<u>1</u>	<u>1</u>	x	x	x	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>		<u>1</u>
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>
<u>1</u>	<u>1</u>	<u>1</u>	x	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>
<u>1</u>	x	x	x	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>
	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>
<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>		<u>1</u>
	<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	
<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>
<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>	

Figure 18: The species shown in Figure 17 can reproduce successfully as a parasite of the species shown in Figure 16 (see text).

							<u>4</u>				<u>4</u>									
			<u>4</u>				<u>4</u>				<u>4</u>				<u>4</u>					
<u>4</u>			<u>4</u>				<u>4</u>				<u>4</u>				<u>4</u>					<u>4</u>

Figure 19: Reproduction of a numerical organism that arose in the experiment of Figure 15 (see text).

									<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>						
							<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>				
			<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>		
<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>	<u>3</u>	

Figure 20: Reproduction of a numerical organism that arose in the experiment of Figure 15 (see text).

						<u>3</u>			<u>3</u>								
			<u>3</u>			<u>3</u>			<u>3</u>			<u>3</u>					
<u>3</u>			<u>3</u>			<u>3</u>			<u>3</u>			<u>3</u>			<u>3</u>		
<u>3</u>			<u>3</u>			<u>3</u>			<u>3</u>			<u>3</u>			<u>3</u>		<u>3</u>

Figure 21: Reproduction of a numerical organism that arose in the experiment of Figure 15 (see text).

								<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>						
						<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>				
				<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>		
		<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	
<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>

Figure 22: Reproduction of a numerical organism that arose in the experiment of Figure 15 (see text).

8 Further research projects

The evolutionary processes observed so far in numerical organisms have been relatively short-lived, lasting no more than a few hundred generations, and have been interrupted by the disappearance of the species. In no case has the beginning of something that could become a unlimited evolutionary process, like that of living organisms on Earth, been observed.

But it is reasonable to expect that under favorable conditions, an evolutionary process that leads to the creation of increasingly viable organisms could lead to the creation of a species viable enough to ensure its further existence. Evolution could then continue indefinitely, or at least as long as the conditions we have established in the “Universe” in which the species evolves persist. This would be the beginning of an unlimited evolutionary process destined to create organisms that are increasingly viable and increasingly adapted to the conditions of the universe in which they live.

The initiation of an unlimited evolutionary process will be the main aim of the research planned for the spring of 1954 to be carried out using Princeton’s electronic computer.

In these experiments, the plan is to employ new rules and place the numerical genes in a two-dimensional universe instead of a one-dimensional one. This would allow each species to interact with a larger number of organisms and deal with a larger range of different situations. In this respect, the conditions would be more similar to those of living organisms on Earth.

9 The living organism

The possibility of producing evolutionary processes based on the same principles that govern biological evolution, while starting from self-reproducing elements of any nature, raises the question of whether numerical organisms, and possibly other types of organisms that can be developed through similar evolutionary processes, should also be considered as living organisms. In theory, such organisms, if developed in universes of sufficient size and under suitable conditions, could have the same and perhaps even greater possibilities for evolution and variety of forms. Whether or not we wish to acknowledge them as living organisms may ultimately be a matter of definition. A very inclusive definition would consider any organism capable of reproduction and undergoing hereditary changes to be living. According to this definition, not only would viruses and genes be alive, but also numerical organisms and the numerical elements from which they are formed.

But nothing prevents us from adding further conditions to our definition, for example, that the conductive elements of heredity (genes) must be molecules or even albumin molecules. In the latter case, we would end up recognizing as living only those organisms created by chemical processes similar in nature to those that created life on Earth.

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